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CS325 Project: Traveling Salesman

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Contents

[Algorithm Analysis 3](#_Toc516252802)

[Greedy Solution: Minimum Spanning Tree (Kruskal’s Algorithm) 3](#_Toc516252803)

[Description 3](#_Toc516252804)

[Pseudocode 3](#_Toc516252805)

[Brute-force Solution 3](#_Toc516252806)

[Description 3](#_Toc516252807)

[Pseudocode 4](#_Toc516252808)

[Exact Solution: “*Branch And Bound”* 4](#_Toc516252809)

[Description 4](#_Toc516252810)

[Pseudocode 6](#_Toc516252811)

[Algorithm Selection 9](#_Toc516252812)

[2-Opt Solution 9](#_Toc516252813)

[Description 9](#_Toc516252814)

[Pseudocode 9](#_Toc516252815)

[Test Results 10](#_Toc516252816)

[Example 1 10](#_Toc516252817)

[Example 2 10](#_Toc516252818)

[Example 3 10](#_Toc516252819)

[Competition Results 10](#_Toc516252820)

[Test Input 1 10](#_Toc516252821)

[3-minute time limit 10](#_Toc516252822)

[Unlimited time 11](#_Toc516252823)

[Test Input 2 11](#_Toc516252824)

[3-minute time limit 11](#_Toc516252825)

[Unlimited time 11](#_Toc516252826)

[Test Input 3 11](#_Toc516252827)

[3-minute time limit 11](#_Toc516252828)

[Unlimited time 11](#_Toc516252829)

[Test Input 4 11](#_Toc516252830)

[3-minute time limit 12](#_Toc516252831)

[Unlimited time 12](#_Toc516252832)

[Test Input 5 12](#_Toc516252833)

[3-minute time limit 12](#_Toc516252834)

[Unlimited time 13](#_Toc516252835)

[Test Input 6 13](#_Toc516252836)

[3-minute time limit 13](#_Toc516252837)

[Unlimited time 16](#_Toc516252838)

[Test Input 7 16](#_Toc516252839)

[3-minute time limit 16](#_Toc516252840)

[Unlimited time 22](#_Toc516252841)

[Bibliography 23](#_Toc516252842)

# Algorithm Analysis

## Greedy Solution: Minimum Spanning Tree (Kruskal’s Algorithm)

### Description

Using Kruskal’s or Prim’s algorithms to find a minimum spanning tree (MST) gives us a greedy solution to the traveling salesman problem, however the solution is only approximate and does not always yield an optimal result. [1] For this method to work, the problem instance has to satisfy the triangle inequality: “for any triangle the sum of the lengths of any two sides must be greater than or equal to the length of the remaining side.” [2] This means that in a graph with several vertices, the shortest distance to travel from vertex A to vertex B is always to travel directly from vertex A to vertex B, and not by visiting any other vertex. This condition is usually satisfied in most instances.

We first must sort the edges of the graph into ascending order and start with the edge that has the lowest weight/cost. Continue examining each edge and select it if it does not form a cycle (unless it is the final edge to complete the Hamiltonian cycle) and does not cause a vertex to have a degree of more than 2. [3] Continue these steps until each vertex has been visited. This method has O(E log E) complexity.

### Pseudocode

Let G= (V,E) be the given graph, with |V| = n {

Start with a graph T= (V,ϕ) consisting of only the vertices of Gand no edges;

Arrange E in the order of increasing costs;

for (i = 1, i ≤ n - 1, i++) {

Select the next smallest cost edge;

if (the edge connects two different connected components)

add the edge to T;

}

}

[4]

## Brute-force Solution

### Description

The brute-force solution is an exact solution, but with an extremely poor runtime. Put simply, given a starting vertex *s*, a brute-force algorithm will look at every potential path beginning and ending at *s*, ultimately returning the smallest path that it encountered. Another way to think of the potential paths is as permutations of the vertices to be visited, with the restriction that the permutations begin and end at *s*. [5]

However, the number of permutations of vertices is extremely large – O(*n*!), where *n* represents the number of cities on the map – so the brute-force algorithm becomes impractical even at an *n* of only 20. [6] Regardless of its impracticality at large values of *n*, the brute-force method is an exact algorithm, so it is worth considering as a baseline for the efficiency and accuracy of our other exact solution, branch and bound (discussed in the subsequent section).

### Pseudocode

The basic pseudocode for the brute-force solution is as follows:

find an initial Hamiltonian tour, called T

set the min tour to T

set the min distance to T.distance

while there are unchecked permutations of T, excluding start and end vertex *s*:

generate a new permutation of T, called T’

if T’.distance < min distance:

set the min tour to T’

set the min distance to T’.distance

[5]

The process of getting permutations is where the complexity is introduced, as there are O(*n*!) permutations of *n* cities. When getting permutations, we would also need to be careful to exclude the start/end vertex *s* from the permutation, since those points will never change. Thus, more specifically, there are (*n* – 1)! permutations of a path from *s* back to *s*, since *s* is excluded from the permutation process. We still need to take into account the distance from *s* to other vertices, though, so we can’t simply remove it from the calculations.

Simple pseudocode for generating the permutations is as follows:

permutations = []

permute(T, start, end):

if start == end:

permutations.add(T)

else:

for i from start to end:

swap(T[start], T[i])

permute(T, start+1, end);

swap(T[start], T[i]) // backtrack

[7]

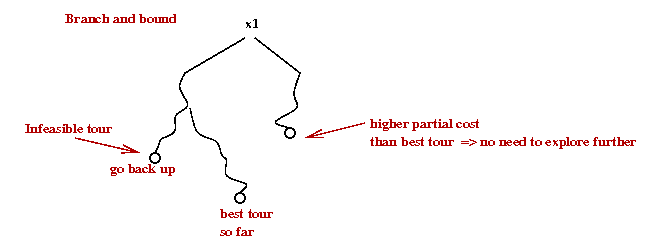
## Exact Solution: “*Branch And Bound”*

### Description

For determining the exact solution, the easiest approach to consider would be that of the “Naïve Approach” or “Brute Force Method” in which case every permutation is examined. A more efficient approach (amongst many) to determining the exact solution however involves either applying a *Branch and Bound* method.

The underlying idea behind the B&B method is that an optimal solution is found by forming a rooted tree of candidate solutions starting at the route node (starting city). From this point, each branch is checked against the upper and lower estimated bounds of the optimal solution, and the branch is discarded if it cannot produce a better solution than the best one found so far. [8]

Below is a simple graphical representation of the B&B method.

[9]

To determine this upper and lower brands, we first need to create an adjacency matrix for the graph in question. After which the matrix is reduced, and we obtain a cost of reduction. For each subsequent node we assign a “cost” value in the following manner:  
(The following matrixes are a direct quote/example taken from: <https://people.eecs.berkeley.edu/~demmel/cs267/assignment4.html>)

i\j 1 2 3 4 5 6 7

\ \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

1 |Inf 3 93 13 33 9 57

2 | 4 Inf 77 42 21 16 34

3 | 45 17 Inf 36 16 28 25

4 | 39 90 80 Inf 56 7 91

5 | 28 46 88 33 Inf 25 57

6 | 3 88 18 46 92 Inf 7

7 | 44 26 33 27 84 39 Inf

[10]

* Reduce this new matrix to obtain an additional cost of reduction

i\j 1 2 3 4 5 6 7

\ \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ Cost of reduction:

1 |Inf 0 83 9 30 6 50 3

2 | 0 Inf 66 37 17 12 26 4

3 | 29 1 Inf 19 0 12 5 16

4 | 32 83 66 Inf 49 0 80 7

5 | 3 21 56 7 Inf 0 28 25

6 | 0 85 8 42 89 Inf 0 3

7 | 18 0 0 0 58 13 Inf 26

Total Cost of Reduction: 84

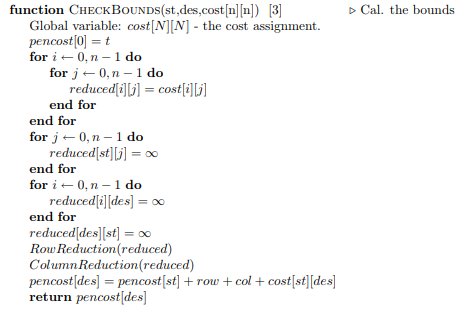
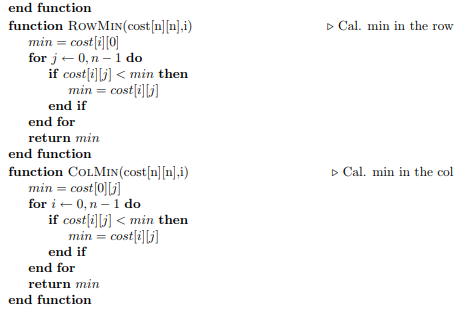
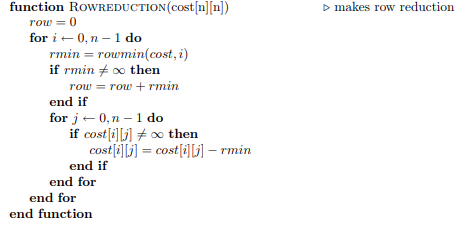
* Add the matrix reduction costs to the path cost C(Root Node, Node-k) + R+ Rk’  
  (Where C() is the distance between two nodes) R is the Root Cost of Reduction, and Rk’ is the cost of reduction of subsequently analyzed node k where subsequent matrix reductions are done per row AND per column.
* Repeat procedure for all connected nodes (obtain the cost of the children nodes, distance + R + R’)
* Select the node with the lowest cost
* Repeat process for that node

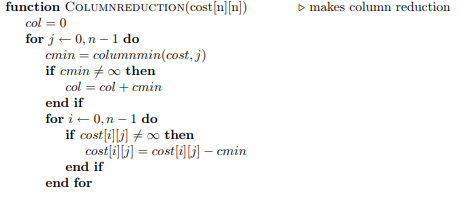
Process is repeated until we have visited all nodes (cities) and thus the optimal TSP path has been found.

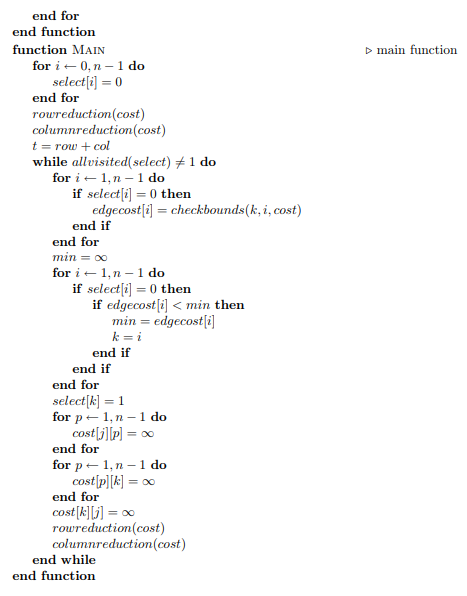
For a much more detailed explanation of the outlined procedure please visit <https://www.youtube.com/watch?v=1FEP_sNb62k> and/or <https://people.eecs.berkeley.edu/~demmel/cs267/assignment4.html>

### Pseudocode

(The following pseudo code is a direct quote from <http://cs.indstate.edu/cpothineni/alg.pdf> ) [11]





# Algorithm Selection

Our group initially chose to implement the branch and bound method to find an exact solution to the traveling salesman problem, however this proved to be difficult. After several days working on the implementation, our program still did not find optimal solutions in the allotted time. Once we had exhaustively combed through the code with no fix, we made the decision to scrap the program and start over from scratch using the 2-opt method.

Although 2-opt is an approximation solution, its results are good enough to be usable for the project, and it has the major benefits of being much faster, easier to understand, and simpler to implement.

## 2-Opt Solution

### Description

The 2-Opt method is a heuristic algorithm that can be used to solve the traveling salesman problem. While it cannot guarantee optimal results for all cases, it can return near-optimal solutions quickly. Two-opt starts with an initial tour and creates permutations by removing two edges from the tour and reconnecting the resulting paths, then evaluates the resulting distance. [12] If the permutation is an improvement, that permutation is kept as the optimal solution. If not, the swap is undone, and the next swap made. Permutations are repeatedly created and compared, until (in our case) 20 consecutive attempts have been made with no improvements. At this point, the best permutation thus far is considered optimal, and the algorithm terminates.

The time complexity of 2-opt is still debated, however the speed can be affected by the method used to generate the initial tour. This can be done using any algorithm, however we chose to use a greedy method, which makes the time complexity O(n2 ln(n)). [13]

### Pseudocode

TSP\_2Opt

while tries < max tries

best distance = tour distance

for i = 0 to (tour size – 1)

for k = i + 1 to tour size

#create new permutation

2\_opt\_swap(i,k)

if new distance < best distance

#reset tries since improved tour found

tries = 0

tour = new tour

best distance = new distance

tries++

2\_opt\_swap(i, k)

take first segment of route (0 to i – 1) and add to the new route in order

take second segment (i to k) and add to new route in reverse order

add remaining portion (k+1 to end) to new route in order

[14]

# Test Results

## Example 1

76 cities

**Length:** 113818

**Tour:** 0 75 2 3 4 6 7 8 9 10 11 12 13 74 14 15 16 17 18 37 36 35 38 39 34 40 41 60 59 61 62 64 73 72 71 63 57 58 56 55 54 43 28 27 26 29 30 19 5 20 31 32 33 42 53 52 51 65 66 49 50 67 70 68 69 47 48 44 45 46 24 25 21 22 23 1

**Time:** 0:00:01.003315

## Example 2

280 cities

**Length:** 2839

**Tour:** 0 1 3 4 5 6 7 9 8 10 11 12 13 14 15 271 16 17 18 24 23 25 22 26 27 28 29 32 33 34 35 36 37 38 39 40 41 42 43 60 61 118 117 115 114 111 110 108 104 90 89 109 112 88 83 82 81 76 84 66 65 85 87 113 86 116 62 63 64 59 44 45 46 47 48 49 50 51 52 53 54 55 56 57 58 68 69 67 70 71 72 73 74 75 77 78 79 80 91 92 93 94 95 96 97 98 99 100 101 102 103 105 107 106 171 172 170 169 168 167 166 165 164 163 162 173 174 161 175 160 159 158 157 119 120 121 122 123 124 125 31 30 126 127 128 21 20 19 133 132 131 130 129 154 155 153 156 152 151 178 177 176 181 183 184 182 180 179 150 149 147 146 145 144 143 142 141 148 139 140 138 137 136 135 134 270 269 268 267 266 265 264 263 262 261 272 273 274 275 276 277 260 259 258 257 256 255 254 253 252 209 208 207 206 205 203 200 199 198 197 193 186 185 187 188 189 190 191 192 194 195 196 201 202 204 213 212 210 211 214 215 216 217 218 219 220 221 222 223 224 225 226 227 228 229 230 251 246 231 232 233 234 235 236 237 238 239 240 241 242 2 243 244 245 247 250 249 248 278 279

**Time:** 0:00:14.440368

## Example 3

15,112 cities

**Length:** a

**Tour:** b

**Time:** c

# Competition Results

## Test Input 1

50 cities

### 3-minute time limit

**Length:** 5592

**Tour:** 0 5 42 49 8 32 30 24 44 16 9 12 38 22 43 6 15 26 7 20 2 47 3 34 25 41 46 23 11 18 39 35 31 27 4 33 19 13 17 48 40 29 37 14 28 36 21 10 45 1

**Time:** 0:00:00.085015

### Unlimited time

***(same as above)***

## Test Input 2

100 cities

### 3-minute time limit

**Length:** 8039

**Tour:** 0 62 20 60 92 40 3 53 79 39 97 80 55 94 30 78 24 95 8 25 87 75 43 6 9 76 91 96 18 99 1 28 63 69 90 14 54 86 98 64 71 45 7 11 49 34 33 29 5 77 35 19 22 83 70 73 88 16 31 46 89 15 38 68 57 61 41 2 23 82 21 4 10 72 56 52 67 47 36 93 44 48 74 66 27 12 32 58 42 59 37 51 17 81 85 26 13 50 65 84

**Time:** 0:00:01.051916

### Unlimited time

***(same as above)***

## Test Input 3

250 cities

### 3-minute time limit

**Length:** 12819

**Tour:** 0 175 7 231 166 75 241 129 45 9 18 91 1 247 52 44 205 198 213 141 233 148 171 151 167 108 62 87 193 172 161 70 199 34 207 214 60 8 211 225 81 114 181 155 182 153 249 83 100 158 206 176 58 243 101 132 165 82 2 55 119 61 49 84 59 32 130 57 232 25 245 221 149 6 135 79 136 98 145 3 115 157 235 238 103 117 226 248 236 227 131 105 152 74 64 159 54 125 154 139 106 23 71 97 65 246 185 121 11 14 184 36 170 76 86 196 31 200 41 43 85 53 146 229 219 144 133 69 47 113 56 5 26 168 240 160 192 195 112 216 104 124 164 163 244 189 147 179 134 178 99 78 126 201 237 28 35 191 68 67 20 215 107 204 111 177 19 17 123 120 94 16 24 188 128 203 96 218 40 37 4 116 77 228 138 33 217 63 122 13 90 234 180 72 109 10 209 223 38 150 88 118 174 137 21 39 42 12 102 80 162 93 27 194 29 156 73 224 230 127 173 208 15 46 95 92 140 202 48 242 169 142 66 30 197 212 190 210 183 220 89 222 186 143 187 239 51 110 22 50

**Time:** 0:00:16.519698

### Unlimited time

***(same as above)***

## Test Input 4

500 cities

### 3-minute time limit

**Length:** 18589

**Tour:** 0 103 376 389 381 39 41 220 72 42 423 385 368 163 293 51 281 256 43 12 291 478 286 354 128 170 203 349 13 57 161 358 189 148 318 329 460 240 415 254 348 283 300 441 35 322 56 175 311 461 238 458 160 453 499 378 104 118 34 233 26 152 374 120 136 289 102 259 404 497 231 164 316 353 242 1 14 215 335 487 373 225 165 344 443 324 294 154 239 411 80 296 456 369 155 477 246 431 345 9 111 397 261 49 65 491 186 101 315 248 350 145 114 20 188 144 486 33 421 339 214 310 413 121 73 16 265 409 303 149 320 494 277 260 100 7 278 328 483 436 79 450 317 204 473 171 403 331 272 342 143 426 463 355 469 112 475 11 249 434 408 390 279 253 479 131 449 251 229 60 336 309 135 439 454 474 217 153 498 195 465 269 209 151 173 482 226 468 332 433 388 159 263 398 462 210 365 230 232 205 367 377 177 95 455 179 305 192 74 295 116 323 162 302 264 412 133 308 48 451 193 108 334 359 395 99 442 27 174 6 94 190 274 219 241 198 496 97 32 495 228 172 363 347 181 298 466 489 372 432 290 422 258 178 180 402 222 109 167 46 129 490 383 418 492 382 270 284 110 176 122 107 337 488 28 257 90 10 29 224 64 185 399 31 266 459 247 30 392 223 206 91 19 24 158 297 429 306 407 142 299 191 168 130 119 444 287 211 375 262 346 366 213 430 401 40 38 184 333 234 292 267 196 425 126 273 22 480 380 391 312 325 150 124 384 452 88 83 420 304 360 78 216 207 476 117 125 137 288 438 326 183 235 55 194 394 319 427 351 405 437 285 313 467 67 440 212 4 156 96 54 218 493 484 362 414 140 419 87 268 446 37 370 50 406 343 197 132 76 371 428 146 69 23 3 243 417 123 199 86 357 59 245 447 330 396 244 113 361 364 138 202 445 424 464 416 157 15 471 147 71 187 386 82 18 77 44 85 166 8 66 448 81 93 2 139 275 379 45 340 485 435 92 106 276 61 52 141 201 271 393 321 75 227 63 255 127 17 307 470 68 301 21 105 89 5 84 169 327 338 250 252 237 200 352 400 410 25 62 236 341 98 47 472 58 356 115 314 208 53 387 280 70 221 481 457 134 36 182 282

**Time:** 0:02:16:601072

### Unlimited time

***(same as above)***

## Test Input 5

1,000 cities

### 3-minute time limit

**Length:** 27757

**Tour:** 0 38 289 297 355 412 457 952 802 580 633 427 293 275 408 239 172 787 621 635 811 136 103 208 738 115 805 515 197 179 101 998 165 620 833 156 677 322 896 49 804 173 973 125 478 745 690 66 946 311 357 748 462 219 105 661 271 747 583 584 127 380 640 650 646 414 953 619 788 597 182 604 404 477 238 113 779 112 214 243 59 161 482 381 181 262 235 27 894 441 439 396 913 875 891 531 949 390 382 902 603 557 140 570 24 629 340 33 536 766 206 192 326 669 932 42 418 610 715 488 823 224 145 255 269 204 174 421 70 78 330 792 13 190 680 415 354 848 615 820 693 512 121 772 306 829 130 424 853 53 273 34 846 718 968 485 599 631 563 151 703 874 578 344 976 67 104 824 212 171 505 11 83 266 403 984 506 342 279 35 986 994 471 840 249 284 183 851 962 167 784 566 431 881 290 992 878 236 257 215 758 979 324 372 68 65 107 307 660 683 622 687 916 783 146 651 300 714 402 995 826 367 756 260 734 377 57 720 593 581 742 359 608 873 497 102 481 941 88 122 456 451 386 797 18 483 655 810 207 812 956 496 452 345 434 218 114 957 437 558 376 705 721 118 368 479 907 270 210 222 612 553 793 333 763 426 389 137 684 682 374 291 109 775 493 480 486 442 908 15 364 544 707 99 337 353 89 538 964 514 865 524 554 883 652 862 276 565 470 837 739 803 422 51 760 903 993 370 7 938 286 722 327 926 8 780 349 755 40 469 170 458 162 10 283 476 551 352 529 299 245 937 2 556 175 44 717 588 319 843 596 855 341 159 589 309 416 567 465 861 607 935 193 800 198 58 825 753 975 274 794 265 943 287 395 664 363 467 81 623 637 110 498 950 813 397 691 801 94 258 980 96 905 149 294 681 925 54 904 501 117 295 969 513 815 939 468 890 460 378 709 743 630 816 503 407 951 808 527 384 87 550 447 736 6 231 694 928 532 438 672 446 176 317 343 126 453 420 166 592 807 298 489 711 312 752 16 605 988 308 445 351 133 858 436 155 186 639 790 768 895 600 100 305 48 871 29 495 37 626 135 60 409 98 590 163 685 534 735 138 234 369 827 69 454 520 450 90 625 180 628 731 918 806 62 216 119 41 95 774 819 141 831 76 316 398 778 997 46 474 844 860 246 318 686 233 657 507 264 884 704 616 209 961 545 487 150 313 494 449 728 697 869 569 91 211 187 832 435 689 252 791 152 915 139 656 880 885 967 947 667 80 428 546 254 472 123 230 47 912 444 670 391 510 22 587 990 868 762 131 533 559 560 845 535 202 906 634 226 781 710 350 85 737 936 571 79 668 74 576 759 143 671 331 188 194 304 3 552 248 767 158 221 999 798 401 358 296 371 433 892 909 636 746 945 323 761 647 900 959 785 17 568 261 872 666 727 128 9 43 511 991 365 225 356 228 719 97 543 240 203 502 411 609 644 392 393 144 841 530 285 974 679 877 278 996 134 282 288 555 910 301 517 63 585 547 432 336 662 542 169 315 39 31 659 314 817 259 854 347 281 955 21 723 303 765 277 256 475 822 548 966 509 863 417 606 373 944 366 821 914 897 72 302 61 751 335 1 160 500 361 106 789 413 852 385 754 129 987 901 272 857 185 537 491 375 649 777 698 911 864 985 712 842 50 461 12 199 920 740 154 764 191 73 726 325 958 348 519 770 564 30 499 632 56 406 263 220 654 849 773 641 981 830 970 157 713 463 280 701 574 52 448 116 948 464 77 733 229 525 443 971 82 25 850 346 627 847 960 601 26 700 749 332 942 699 886 20 618 184 4 814 866 856 696 5 729 834 178 809 132 484 201 887 611 540 963 665 522 394 867 200 518 466 898 19 142 440 93 241 124 328 196 388 177 965 870 387 893 405 648 594 614 617 675 977 523 591 795 933 586 516 28 922 247 111 310 526 663 383 828 818 730 164 379 267 205 638 250 889 410 455 473 562 983 978 573 71 362 924 232 921 702 658 642 217 577 338 899 268 836 572 744 430 423 595 400 769 108 954 575 195 32 678 253 425 624 688 972 521 549 86 929 223 237 242 329 539 504 148 419 989 120 725 613 706 321 771 692 888 14 838 399 931 147 839 796 882 876 982 879 320 927 917 757 676 528 673 732 84 360 492 674 213 153 64 859 598 55 934 786 930 750 741 541 653 339 645 244 45 189 251 490 695 508 776 92 227 799 459 724 923 292 835 561 579 23 75 716 429 36 708 334 582 782 602 940 643 168 919

**Time:** 0:02:55:65681 *(cut off by internal timer)*

### Unlimited time

**Length:** a

**Tour:** b

**Time:** 0:17:25.085505

## Test Input 6

2,000 cities

### 3-minute time limit

**Length:** 41299

**Tour:** 0 1263 387 1076 1297 1113 1611 1074 1447 396 1775 620 51 1624 326 416 1629 102 1090 633 1412 264 1540 196 800 1926 944 1256 513 1010 415 199 83 1486 636 548 194 1976 297 1505 1958 1638 368 1124 52 1602 1571 1251 1314 1172 373 1363 1628 1127 1454 1145 1069 1795 1023 1352 1504 262 1752 35 1797 1812 1726 1170 4 258 62 106 1669 600 1524 1336 1982 1613 1083 847 252 1489 1779 770 773 669 1743 118 1391 1637 277 239 1012 6 1137 1873 1919 1118 1640 1375 877 1612 43 552 1207 1068 941 97 646 470 1255 1057 1817 1494 562 1159 1247 1857 1851 572 1830 342 642 1286 88 654 1903 1693 740 604 193 787 152 288 225 1410 1318 1217 780 333 676 17 908 573 1481 828 673 955 540 990 1677 1165 709 399 1249 269 1852 1938 691 1342 1648 1531 1719 1794 1425 1271 712 1324 488 1841 184 1788 104 331 1798 160 883 672 1320 1700 250 281 1969 592 1813 1641 622 1560 760 1250 891 899 952 435 1093 896 182 155 1626 1932 1488 569 895 1605 3 1606 924 982 1202 479 1666 992 374 634 238 1227 1183 1826 1464 1916 1931 446 1674 1835 1946 371 1008 1175 1499 1988 1373 517 1542 960 127 799 335 438 224 1230 1234 1378 1924 532 1948 917 1365 1044 1620 1487 1696 870 614 1721 499 832 165 498 1422 735 1865 1053 1408 1753 237 1749 1284 711 1618 1335 1383 518 1349 1575 1600 626 1507 796 1850 759 1722 1653 615 1877 408 432 567 1875 1061 1573 161 1399 738 457 1803 919 1066 660 861 515 1164 1870 1633 589 972 544 1780 1358 409 359 159 268 455 789 228 1847 602 1861 183 1417 590 1474 1513 594 1404 87 354 867 1623 245 1594 1135 10 979 380 1433 1453 452 1366 1906 1905 1043 835 1086 825 116 320 1224 1998 935 340 813 1429 1597 1482 767 1033 430 1825 668 76 91 79 142 1350 868 581 1273 463 804 907 1438 1994 528 392 1158 754 95 1005 1565 915 85 826 951 988 1801 232 1346 1744 1307 591 338 125 1889 314 207 744 508 1024 73 1128 1037 1644 1583 1572 334 1295 954 111 1576 949 1974 381 143 1246 1848 890 937 1519 772 976 364 31 78 670 282 1131 1717 1111 1268 1502 1385 1214 1394 812 1305 280 436 1173 1002 1177 1218 683 1836 1898 307 1317 606 1468 343 1151 1781 814 162 1348 783 587 137 1510 535 367 1777 1088 210 47 1106 715 163 1829 1442 999 1959 681 1211 349 527 700 1392 32 1262 350 63 1712 1821 756 843 325 407 1209 1334 649 651 46 1704 1785 1126 317 1843 313 757 1107 1791 611 733 693 764 1957 1229 482 623 1541 748 989 1671 92 1621 657 30 993 679 1161 995 1141 921 1178 1622 957 1893 1563 1853 453 1616 1071 939 1498 1860 923 1564 884 424 1676 1928 1515 201 1327 561 26 1546 1139 171 737 1806 695 1243 753 1300 311 100 1786 706 185 433 566 117 429 1465 1384 1 1081 507 900 1221 14 1954 805 1947 1253 1547 1990 963 254 147 75 164 1427 490 563 82 1030 707 932 730 1027 538 869 853 769 727 749 1484 244 1533 421 1736 394 1463 1970 1636 89 842 121 1237 1766 621 1045 1937 1762 1092 1193 1920 1094 1585 998 794 1934 627 1718 377 1733 1492 1754 1617 1838 579 1395 186 1319 1435 1370 1296 1338 267 866 1313 1459 1157 1811 365 879 559 1742 1174 1288 58 820 1070 692 234 477 1632 1935 1149 1129 542 1986 173 1344 553 1180 584 746 1846 1630 1260 1649 1109 103 1882 1194 449 450 1216 1001 1804 180 138 1814 294 1699 888 434 1555 958 533 16 1771 248 1663 445 2 1871 1756 188 1432 1476 451 609 1244 427 852 1900 1155 240 38 1751 398 1584 310 285 1831 214 815 903 1374 1238 1845 1713 784 522 1715 1668 793 1266 419 1122 724 845 231 1414 348 986 821 1143 1110 1807 1325 1561 603 273 568 1820 1914 1026 1041 897 1883 1999 1242 871 1312 1241 1979 1446 1701 1267 1195 854 1409 1739 243 966 157 929 1058 585 806 1360 849 665 485 1052 583 1529 1657 363 1965 437 1890 328 1759 790 1953 1655 93 290 1518 726 1133 1148 1000 1015 1590 1796 1406 402 525 200 352 126 625 420 1951 1580 1382 473 1475 1964 1096 1368 1651 1191 1208 1558 1047 1449 1333 586 1579 1895 275 1168 1444 1562 1424 1469 90 1100 1685 1407 1282 1065 575 67 920 968 1108 694 176 316 739 596 1810 287 37 1711 702 323 469 355 601 1802 1854 1709 1125 703 543 1940 1520 1656 131 18 1856 169 985 1627 637 1684 892 1095 1372 640 1292 593 1918 1809 1146 1968 422 526 1289 565 1966 462 332 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**Time:** 0:02:55:643069 *(cut off by internal timer)*

### Unlimited time

**Length:** a

**Tour:** b

**Time:** c

## Test Input 7

5,000 cities

### 3-minute time limit

**Length:** 64229

**Tour:** 0 3302 2667 1954 4107 2601 212 3833 3996 868 3272 2975 223 1192 2711 2302 4512 4386 1335 1201 3616 244 1668 1409 335 1010 3167 1568 2026 4858 133 1021 3678 4588 2099 3947 3179 3888 3636 3325 1526 2489 1229 1162 1103 3620 2335 4143 2330 3707 4563 2835 226 4231 4021 4891 3871 2594 2590 3630 1485 4381 1862 2511 649 56 2672 2660 2346 1977 505 810 2889 774 3087 1305 1459 1528 1638 2432 1406 1946 1295 4640 3687 1152 4287 1567 3025 621 2163 1667 1725 2158 1250 2439 2382 1117 2117 2270 970 563 766 4843 2385 3342 4919 4427 3787 581 286 628 592 4672 666 3696 1674 3451 3258 4890 1379 283 1904 4962 4923 4320 236 2798 3885 4736 4070 3783 702 4127 630 3472 2821 2359 2442 2709 4480 329 3746 535 1260 3836 2942 3372 254 1173 4466 1159 4870 4707 1481 4576 1150 4925 2005 540 3257 2238 2959 3136 3602 3512 887 399 2668 1228 3232 4664 121 3182 1982 279 4048 3571 4924 3770 1203 3239 4158 1866 4303 2056 943 3777 1846 3267 4061 4277 2068 2715 4862 1160 3962 2241 1775 1758 2611 3587 4730 2139 3505 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**Time:** 0:02:55:196969 *(cut off by internal timer)*

### Unlimited time

**Length:** a

**Tour:** b

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